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THE WORLD TRADE WEB: STRUCTURE, EVOLUTION AND MODELING

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Contents

Summary

The World Trade Web (WTW) is the complex network representation of the international trade system that allows an analysis at the large scale from an interdisciplinary approach. Countries are represented as nodes and commercial relations between them as links. The network representation offers a new level of description that goes beyond the country-specific analyses used in more traditional economic studies of trade. In particular, it makes possible the analysis of the indirect trade interactions among world countries. In this line of research, several tools and methodologies that have been recently developed for the analysis of any type of networks can be exploited to extract information from the WTW, and to discriminate which

properties signal a nontrivial structural organization and which are likely to be originated by chance or structural constraints. Although these results have been obtained recently, and during a relatively short period of time, they have already established various robust empirical signatures of the international trade network. In some cases, these "stylized facts" turn out to be stable in time, while in others they highlight previously unrecognized changes in the system. We present a self-contained description of these advances. After a general introduction to the international trade system, we describe the possible representations of the WTW that have appeared in the literature, from its purely topological properties to its weighted structure and directionality, and the added levels of understanding they convey. We then describe various models that have been proposed to reproduce the empirical properties of the WTW, from more traditional "gravity models" which predict expected trade volumes but cannot reproduce the topology of the web, to recent network-inspired models that succeed in explaining the observed complexity of the network at a topological level. We finally discuss some open questions for future research on the world trade system as a complex network.

1. Introduction

Human societies are complex systems of individuals organized in different characteristic infrastructures. All of those, from cultural or political to scientific or economic, are interrelated forming a conglomerate that takes collective behavior beyond a mere superposition of individual activities. However, we are just starting to understand the need to develop techniques and methodologies that would allow us the necessary understanding of societies as a whole. For the moment, reductionist approaches which isolate specific social structures or patterns of interaction -complex enough on their own- dominate and have produced the valuable results we have at hand today.

In this compartmentalized panorama, one of the fundamental infrastructures of social organization is economy. It includes all systems of production, exchange and consumption of goods and services within local regions, or between countries in the world or other supranational areas. Many factors, including history, level of technological development and wealth, geographical location or political treaties impact modern economies, which specific elements are commonly aggregated in two separate blocks: the real economy, that concerns labor, production or trade, and finance, that includes debt and investment. There is a vast literature covering both fields, from the seminal works of Adam Smith in the eighteenth century to the ideas in the twentieth century of John M. Keynes and the post-Keynesians.

Zooming in on the basic mechanisms of the real economy, a fundamental form of economic interaction is trade that in most countries represents a significant share of their gross domestic product (GDP). At the international level, countries exchange goods and services usually through the structure of a market. Economies import or buy assets from other countries and at the same time export or sell to them. These transfers require a transmission infrastructure and have grown parallel to the development of communication between human groups from prehistoric times. Both have evolved boosted by waves of globalization, periods when a complex series of closely intertwined changes have dramatically increased the interactions and interdependences between an increasing number of people and human organizations in disparate geographic regions. These changes, when applied to the international economic order, refer to the presence of an intricate network of economic partnership among an increasing number of countries.

In terms of trade, two waves of globalization are identified in recent history and correspond to processes of decolonization and breaking of technical barriers inducing downturns in costs and time expenditures. The first wave is roughly identified from 1870 until the beginning of World War I and was related to lowered costs for transportation of materials and goods triggered by the Industrial Revolution, with steam power encouraging the expansion of railroad networks and oceanic routes and the telegraph connecting the two sides of the Atlantic. The second wave, from 1960 to the present -there is no complete agreement whether a third, middle wave has occurred- is intimately related to ease of exchange of information and ideas facilitated by the Information Technology Revolution, which is causing communications costs to drop dramatically at the same time that information management capabilities are exploding.

As a result of those globalization processes, the large-scale organization of the world economies exhibits nowadays a high level of local heterogeneity and of global interdependency at the same time. In this scenario, the relevance of trade goes beyond a mere exchange of goods and services. On the one hand, commercial trade flows are indeed highly correlated with other types of cross-country economic interactions -flow of services, financial assets, workers, etc.- and so stand as a good indicator for more general economic relations. On the other, and leaving aside technological, cultural and other non-economic social aspects that interplay with trade, feedback mechanisms operate between international trade and other economic variables such as production, investment, debt, or currency prices. Trade plays a central role as one of the most important interaction channels between states. It can act, for instance, as a substrate for the transmission of economic policies, cycles, and shocks like the 1997 Asiatic crisis, which shows how economic perturbations originated in a single country can somehow propagate globally in the world. This is in line with the idea of the world becoming a global village. In a broad sense, this implies that the collapse of the barriers in human communication allows that incidences in one part of the system affect all the rest.

Therefore, it seems natural to analyze the international trade system from a global perspective taking into account every country and its trade relationships regardless of its size or wealth. This is in contrast to traditional analyses of international commerce that have been based on local approaches. These have focused, with a few exceptions, on bilateral trade exchanges, commercial relations between pairs of countries. However, the large size and the entangled connectivity pattern characterizing the international trade organization points out to a complex system, whose properties depend on its global structure. Although economic or political institutions can have an impact in local regions, at the large scale the world trade web resembles other complex self-organized systems, which evolve without the intervention of any centralized control that regulates its growth or performance. At this scale, an integrative interdisciplinary framework coming from complex networks science considers the set of all exchanges in the system as a whole and has proven to be successful in uncovering the relation between local and global emerging features and in providing insight into some of the global properties and evolution of the international trade system.

Within the complex network representation, countries correspond to nodes and trade relationships among them to links. As a tool of visualization, graphs of bilateral trade relations have been used in recent years to help analyze gravity models, often proposed to account for the world trade patterns and their evolution. However, the importance of the complex network approach goes beyond the auxiliary character of visual representations and introduces new interdisciplinary methodologies for the analysis of the world trade web at the large scale. As specific examples, the effects of one economy on another can be assessed more reliably based on the complete set of complex interactions that interwove the whole system, and correlations between the income of countries measured by their Gross Domestic Product (GDP) and their role as net producers or net consumers can be explored. From a more general perspective, the complex network approach suggests that the evolution of the WTW is guided by collective phenomena, and that self-organization plays a crucial role in structuring its heterogeneities and its hierarchical architecture.

The next sections present a review of the world trade web as a complex network. Various representations of the network are possible. The simplest approach is to consider it as undirected, which amounts to ignore the directionality of trade. This representation can either discard the volume of trade, in which case one has an unweighted undirected network, or take trade volume into account, so that a weighted undirected network is obtained. Such undirected approaches already reveal a non-trivial and heterogeneous organization, which is discussed in Section 2. More refined descriptions focus on the direction of trade. In the unweighted case, this highlights peculiar patterns in the reciprocity of trade relationships. In the weighted case, the directionality of exchanges reveals a strong heterogeneity in the magnitude of the different bilateral trade relations and their asymmetry. Directed approaches are reviewed in Section 3. In Section 4 we present a series of attempts that have been made to model the observed properties of the network. These are essential issues in the understanding of the interplay between the underlying structure and the principles that rule the functional organization and evolution of the world trade web. Finally, we list some open questions for future research on the international trade network in Section 5 and make our concluding remarks in Section 6.

2. The Undirected Network Approach

Complex networks are ubiquitous in nature and among manmade systems. Examples range from intra-cell networks –gene regulatory, metabolic, protein interaction, signal transduction networks…- or inter-cell networks –nervous systems and brains, tissues- where cell functionality is sustained by the network structure, to technological webs –the Internet, the world wide web, wireless communication networks…- where topology determines the system's ability to transmit information, or to social networks of interacting individuals. Over the last years, complex networks have been the subject of an intense research activity both on empirical and theoretical grounds that have set the grounds for complex network science.

Complex networks are structures of a large number of elements linked by nonlinear interactions that self-organize and give place to emerging phenomena. Real networks are not regular lattices neither completely random. Their structure lies in between and most of them share a set of universal topological features despite belonging to very different domains. Typically, networks show the following properties: scale-free (SF) degree distribution $P(k) \sim k^{-\gamma}$ with $2 < \gamma \leq 3$, where the degree k is defined as the number of nearest neighbors of a node; the small-world property, which states that the average path length between any pair of vertices counted as the number of intermediate neighbors grows at most logarithmically with the system size; and a high clustering coefficient, that is, the neighbors of a given vertex are interconnected with high probability; in addition, degree-degree correlations that account for the probability that a vertex of degree k is connected to a vertex of degree k' and is said assortative if highly connected vertices tend to attach to other highly connected vertices (characteristic of social networks, such as scientific collaboration networks), and disassortative if conversely highly connected vertices tend to attach to poorly connected ones (technological networks such as the Internet).

As we describe below, the empirical characterization of the international trade system as a graph built upon the trade relationships between different countries in the world displays the typical properties of most complex networks. It has a broad degree distribution, where most countries have a low number of connections while just a few trade with nearly all the system; it is a small-world, so that each pair of countries is very close in topological distance; it displays a decreasing degree-dependent clustering coefficient, signaling that countries trading with a well connected one are poorly interconnected among them, and it presents degree-degree correlations between different vertices, with high-degree countries connecting preferentially to low-degree ones. All these properties make the world trade web a complex network, which is far from being well described through a classical description.

2.1. Topological Features

The WTW is customarily constructed from databases (see Section I in

Appendix) detailing the import and export exchanges of merchandizes between pairs of countries in the world. Imports correspond to goods brought into one country typically from being bought to another country, and provide domestic consumers with foreign production. In its turn, exports correspond to goods from one country brought into another country typically from being sold, and provide foreign consumers with domestic production. Therefore, exports render an inflow of money into the country while imports generate an outflow. Notice that the same commercial exchange between two countries would be at the same time an importation for one of them while an export for the other, and that one country can simultaneously have import and export exchanges with another.

2.1.1. Number of Countries in the World Trade System

The number of countries in the world trade system has been increasing during the last century (see Figure 1). The progressive crumbling of the Imperial Colonies during the sixties and seventies, and the former Soviet Union could basically explain this fact. However, as shown in the inset, the density of connections has remained constant, which indicates that the average number of trade partners has grown, during the same period, linearly with the number of countries. This can be understood by assuming that, when a country splits, the different offsprings usually maintain a fraction of the trade relationships of the former unit. This mechanism implies that the evolution of the number of connections evolves according to the equation

$$\frac{dE}{dN} = \alpha \langle k(N) \rangle, \qquad (1)$$

where E is the total number of trade relationships in the system when its size in number of countries is N and $\langle k(N) \rangle$ is the average number of trade partners of a country at the same moment in time. Since, by definition, $\langle k(N) \rangle = 2E/N$, we obtain that $E \sim N^2$ and, thus $\langle k(N) \rangle \sim N$, in accordance to the empirical observations.

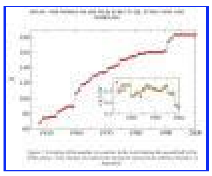

Figure 1. Evolution of the number of countries in the world during the second half of the XXth century. Inset: density of connections during the same period. (DBII in Section I of Appendix).

2.1.2. Network Reconstruction

The empirical data allow graph reconstructions of the international trade system with different levels of detail and information content. In general, countries are represented as nodes and edges join pairs of nodes based on some metric related to trade. The most basic representation is a graph where a pair is connected through an undirected and unweighted link whenever a commercial exchange between them is reported. More sophisticated representations take also into consideration the amount of the exchange, so that links are not anymore binary, just present or absent, but have an associated weight that introduces a new level of heterogeneity in the picture. On top of this, the directionality of the connections can also be taken into account to consider links with different orientation and/or different incoming and outgoing weights, or purely directional weighted connections corresponding to net trade imbalances.

Despite its simplicity, the most basic representation of the WTW as an undirected unweighted graph already provides relevant information about the international trade system. The adjacency matrix $\mathbf{A}$ that compiles the connectivity information between nodes in the network has entries a_{ij} with values 1 or 0, depending respectively on whether country i has or not an exchange with country j. The dual attribute of trade exchanges, that at the same time are imports for one country while exports for the other, can be exploited to reconstruct a consistent unweighted undirected adjacency matrix $\mathbf{A}$ from the import and export databases. In mathematical terms $I_{ij} = E_{ji}$, where $\mathbf{I}$ is the import adjacency matrix with entry I_{ij} equal to 1 if i imports from j and 0 otherwise, and $\mathbf{E}$ the export adjacency matrix with E_{ij} equal to 1 if country i exports to country j. The adjacency matrix $\mathbf{A}$ is calculated as

$$a_{ij} = \frac{I_{ij} + E_{ji}}{1 + \delta_{I_{ij}+E_{ji},2}}, \qquad (2)$$

where $\delta_{x,y}$ is the Kronecker delta function. This method enables to obtain an adjacency matrix where each connection is relevant at least to one of the two involved countries, even when only partial information is reported, as in the case of a bounded number of merchandizes. An alternative reconstruction of the undirected adjacency matrix would consider just bidirectional edges, but in the following we consider $\mathbf{A}$ as defined in Eq. (2).

2.1.3. Unweighted Topology

The adjacency matrix A encodes all the relevant topological information of a network. However, it involves a large number of variables that makes it quite unmanageable. In order to extract the characteristic features of the network, information is usually recovered and coarse-grained from it in other more handy scalars or uniparametric functions. The average degree $\langle k \rangle$ is defined as the average number of connections per node and along with the number of nodes, N, gives an idea about the density of connections in the network. For the year 2000 and DBI (see Appendix 1), it has been reported to be $\langle k \rangle = 43$, that is a very high average number of trade partners for $N = 179$. Another scalar measure is the average shortest path length $\overline{l} = 1.8$, which measures the average distance in number of connections between any pair of nodes. This value is very low indicating that pairs of nodes are on average very close topologically and is very similar to the one expected for a random network of the same size and average degree.

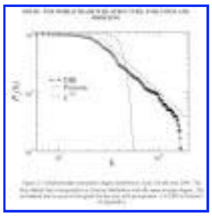

Figure 2. Complementary cumulative degree distribution $P_c(k)$ for the year 2000. The blue dashed line corresponds to a Poisson distribution with the same average degree. The red dashed line is a power law guide for the eyes with an exponent -1.6 (DBI in Section I of Appendix).

More detailed information is provided by the degree distribution $P(k)$. This quantity measures the probability of a randomly chosen vertex to have k connections to other vertices. The complementary cumulative distribution of the WTW, defined as $P_c(k) = \sum_{k'=k} P(k')$, shows a flat approach to the origin (see Figure 2), indicating the presence of a maximum in the degree distribution $P(k)$ around $k \sim 20$, similar to what is found in the Erdös-Rényi network. However, for larger values of the degree (but smaller than the number of countries) the complementary cumulative distribution displays a slow decay. A power-law fit of the form $P_c(k) \propto k^{1-\gamma}$ yields $\gamma = 2.6$, showing a strong deviation from the fast decaying tail predicted by the classical random graph theory. Such a value of the exponent γ is within the range of SF networks, for which the second moment of the degree distribution diverges. This SF property implies an extremely high level of degree heterogeneity, with most nodes characterized by a lower number of connections while a small number of countries display a very large number of neighbors. Nevertheless, in the WTW the degree distribution displays a sharp cut-off restricting the SF behavior to a very narrow region above which the degree of many vertices “saturates” to values close to the number N of world countries. This is an unusual property, not observed in other real networks, which signals that some countries almost approach the maximum allowed connectivity and render the WTW extremely dense. By contrast, sparse networks are characterized by a maximum degree much smaller than the total number of vertices.

The explanation could be found in a difference in the mechanisms for the formation of the WTW, which could differ depending on the particular political and economic situation of a country. Low-degree countries, most of which turn out to be the poorest, are basically constrained to subsistence trade flows and, therefore, preferential attachment mechanisms that have been proposed to explain SF degree distributions could not hold. As expected, there exists a positive correlation between the number of trade channels of a country and its wealth, measured by the per capita GDP. This correlation is found to be high, 0.65, which means that, indeed, most low-connected countries are poor countries—Angola, Somalia, Rwanda, Cambodia . . . —and most high-connected countries are rich countries—the USA, Japan, Germany, and the UK, for example. However, there also exist a significant number of cases in the reversal situation, that is, high per capita GDP countries with a small number of connections and low per capita GDP countries with a large number of trade channels. A germane example for the first circumstance is that of Norway and Iceland which are among the top ten wealthier countries but only have 56 and 24 trade channels, respectively. For the second case, Brazil, China, and Russia are typical examples. Remarkably, these results change substantially if one considers total GDP instead of per capita GDP. In this case the correlation is even stronger and, even if some variability is still present, the general trend is a steadily increasing relation between total GDP and degree. For the year 1995, this trend is shown in Figure 3, where the degree k_i of every country i is plotted against the rescaled variable x_i, defined as the ratio between the total GDP of country i and the average of the total GDP over all countries. As we describe later on when considering models of the WTW, the dependence of the structural properties of the network on the total GDP goes even further. Indeed, the knowledge of the GDP of all world countries allows to predict the probability that two countries trade, and also the expected traded volumes. Many observed properties of the WTW, not only the degrees, can therefore be predicted as a function of total GDP, plus in some cases a few other factors that we illustrate later on in Section IV.

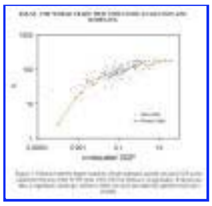

Figure 3. Relation between degree (number of trade partners) and the rescaled GDP in the undirected version of the WTW (year 1995, DBII in Section I of Appendix). Besides raw data, a logarithmic binning is shown to filter out noise and make the general trend more evident.

Information about degree correlations in the network is provided by the average nearest neighbor degree and the degree dependent clustering coefficient. Two-point degree correlations are measured by the conditional probability $P(k'|k)$, which gives the probability of a vertex of degree k to be linked to a vertex of degree k'. This function is difficult to treat due to statistical fluctuations. To characterize this correlation, it is more useful to work with the average nearest neighbors' degree, defined as

$$\bar{k}_{nn}(k) = \sum_{k'} k' P(k'|k) = \frac{1}{N_k} \sum_{i \in \nu(k)} \frac{1}{k} \sum_j a_{ij} k_j, \qquad (3)$$

where N_k counts the number of nodes of degree k and $\nu(k)$ is the subset of nodes with this degree. For uncorrelated networks, this function is independent of the degree k, but for the WTW it displays a clear dependency on the vertex's degree. In some region of the degree domain, the curve can be fitted with a power law decay $\bar{k}_{nn}(k) \sim k^{-\nu}$ with $\nu \sim 0.5$ (see Figure 4). This result means that the WTW is a disassortative network where highly connected vertices tends to connect to poorly connected vertices. This is in agreement to technological networks such as the Internet and other competitive systems, but in contrast to social networks, which tend to be assortative.

Regarding three-point correlations, the local clustering coefficient of a vertex of degree k_i is defined as $c_i = 2n/[k_i(k_i - 1)]$, where n is the number of its neighbors that are interconnected between them, so that c is a measure of the number of triangles attached to a node. A uniparametric function of the degree can be defined by averaging c_i for all nodes in the same degree class $i \in \nu(k)$. This degree-dependent clustering coefficient can be formally expressed as a function of the probability $P(k', k'' | k)$ that a node of degree k is connected to two nodes of degrees k' and k'', and of the probability $p_{k'k''}$ that these two nodes are connected between them, that is,

$$\bar{c}(k) = \sum_{k'} P(k'k'' | k) p_{k'k''} = \frac{1}{N_k} \sum_{i \in \nu(k)} \frac{1}{k(k-1)} \sum_j a_{ij} a_{il} a_{jl}. \quad (4)$$

In uncorrelated networks this quantity does not depend on the degree of the node but, again, for the WTW there is a region where it shows a power law behavior of the form $c(k) \sim k^{-\mu}$, with $\mu \sim 0.7$ (see Figure 4). The clustering coefficient averaged over the whole network is $C = 0.65$, greater by a factor 2.7 than the value corresponding to a random network of the same size. This result, together with the scaling law for two-point correlations reveals a hierarchical architecture of highly interconnected countries that belong to influential areas that, in turn, connect to other influential areas through hubs.

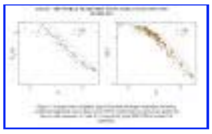

Figure 4. Average nearest neighbor degree (left plot) and degree-dependent clustering coefficient (right plot) versus degree in the WTW. Dashed lines are power law guides for the eye with exponents -0.5 and -0.7, respectively (year 2000, DBI in Section I of Appendix).

The hierarchical structure of a complex network can also be detected through another important degree correlation measure, the rich club coefficient. Heterogeneous networks have far from uniform structural properties and show an extreme dispersion that marks a few of their elements with the highest values as dominants. This is the case of hubs, nodes with the highest degrees. In what manner these top elements relate to each other, in particular, whether they are polarized or, on the contrary, show a tendency to club-forming elites or backbones, is analyzed as the rich-club phenomenon.

To detect if hubs aggregate in well interconnected cores, the rich-club coefficient, a uniparametric measure as a function of the degree k, was proposed as the fraction of edges actually connecting nodes with degree larger than a certain threshold out of the maximum number of connections if they formed a perfect clique, $\varphi(k) = E_{>k} / [N_{>k}(N_{>k} - 1)]$. Later on, the original metric was redesigned by introducing a comparison to the corresponding value in the randomized version of the graph that preserves the degree distribution $P(k)$, $\rho(k) = E_{>k} / E_{>k}^{\mathrm{ran}}$, in order to discount structural effects forcing hubs to be connected without the intervention of special ordering principles. In this way, rich clubs have been found in scientific collaboration networks and in critical infrastructures such as the world air transportation system. In the case of the world trade web, structural effects – the small size of the system combined with a high density of connections heterogeneously distributed among the nodes- dominate and, therefore, an observed high level of interconnection between the hubs does not manifest a tendency towards forming economical elites but it is just due to topological constraints. Repulsion between them is neither observed, despite being expected in highly competitive systems. In the international trade system, economies compete for new markets and, in particular between the hubs, it would not be expected more collaboration than strictly necessary to keep the global organization connected and functional. Nevertheless, as we show in the next section, by considering weights in the connections, the rich club analysis reveals unexpected and interesting patterns.

2.2. Weighted Structure

Search Print this chapter Cite this chapter

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2.2. Weighted Structure

The first approximation that represents the unweighted description of the WTW taking the interactions between pairs of elements as binary, just present or absent, turns out to be an oversimplification that in some cases can distort the interpretation of the results. A weighted network approach, that incorporates the information about the intensity of trade interactions, can greatly improve the descriptive power of the topological representation of the WTW. In this description of the international trade system, interactions are quantified by their intensity or weight w, and the nodes have associated strengths s giving the actual magnitude of all the interactions handled and defined as the sum of the weights on the links attached to the nodes. Weights usually are given as the annual merchandise exchanges in some specific units such as current year US dollars, or other units that for some studies can conveniently be adjusted for inflation.

As for the purely topological description, one can perform either a directed or an undirected analysis of the network. In this section we describe the results obtained when an undirected weighted representation is adopted. The directed weighted approach will be discussed in the next section. If weighted links are considered undirected, then for consistency reasons the weights defined on them should be symmetrised, i.e. the weight w_{ij} must equal the symmetric quantity w_{ij}. One way to do this is to define weights as the total trade exchange between pairs of countries, so that w_{ij} represents the volume of trade from country i to country j plus the volume of trade from country j to country i. Alternatively, one could define w_{ij} as the average value of the two flows, which simply introduces a factor ½ in the above definition. With this prescription, the distribution of link weights has been found in some studies to approximately follow a log-normal shape, which remains robust over a period of 53 years. Another analysis reports a distribution that is slowly moving from a log-normal density towards a power law in the last 20 years. In both cases the intensity of trade exchanges shows strong heterogeneity. In the same way as a small number of countries trade with almost every one else while the majority have exchanges with a much reduced set of neighbors, the majority of existing links are associated to weaker trade relationships as compared to the largest values in the network.

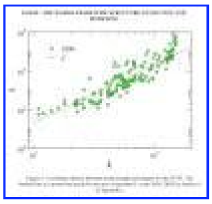

Figure 5. Non-linear relation between node strength and degree for the WTW. The dashed line is a power law guide for the eye of exponent 3. (year 1990, DBIII in Section I of Appendix).

Regarding degree correlations in the weighted version of the WTW, it has been observed (DBII, year 2000) that two- and three-point correlations measured as

$$\bar{k}^{w}_{nn}(k) = \frac{1}{N_k} \sum_{i \in \nu(k)} \frac{1}{s_i} \sum_j w_{ij} k_j \qquad (5)$$

and

$$\bar{c}^{\,w}(k) = \frac{1}{N_k} \sum_{i \in \nu(k)} \frac{1}{s_i^2 (1 - Y_i)} \sum_j w_{ij} w_{il} a_{jl},$$

(6)

respectively, are extremely weak or close to flat (see Figure 6). This suggests that the understanding of their formation processes or their modeling can be simplified by avoiding correlations at the weighted level. The behavior of the weighted correlation measures is in contrast to the observed for the binary representation, where it is found to decrease as a function of the degree, as it can be observed in Figure 6. In general, for networks with a nonlinear relation between strength and degree, like the WTW (see for instance Figure 5), weighted measures greatly disagree with the unweighted ones, offering a completely different picture with respect to the bare topology.

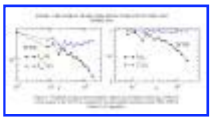

Figure 6. Weighted average nearest neighbor' degree and weighted clustering coefficient versus degree in the WTW as compared to the unweighted measures (year 2000, DBII in Section I of Appendix).

Even though the previous correlation functions indicate that hubs in the weighted world trade web tend to be more clustered as compared to the binary version, this does not mean that they do it purposively to form oligarchies. On the contrary, they indeed form a multipolarised set in agreement with the fact that the world trade web represents a highly competitive system. This can be seen from measuring the weighted rich-club coefficient. One can focus exclusively on strengths and assume that the rich nodes are those with the highest strengths. The thresholding procedure for selecting nodes in the weighted network with strengths larger than a certain value generates then a hierarchy of nested subgraphs, and the sum of the weights on the links within each subgraph, $W_{>s}$, is considered in order to calculate the weighted rich-club coefficient. Regarding the normalization, there are different ways to compute what is expected for a maximally random case. A possibility is to compare to a randomized version of the graph that preserves the strength distribution $P(s)$. This null model can be achieved by approximating the weights in the network by integers, so that they could be considered as multiple connections formed by decoupled links. Then, the randomization based on rewirings −two links are selected independently at random and their ends swapped-- can be done, avoiding self-connections but not multiple ones. In this way, the nodes maintain their strength but the weights in the links (or the degrees) can change in the process. Formally, the rich-club coefficient in the weighted approach under the null model described above can be written as $\rho(s) = W_{>s} / W_{>s}^{ran}$.

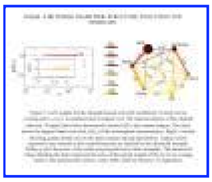

Figure 7. Left: graphs for the strength-based rich-club coefficient. Dotted curves correspond to $\rho(s_T)$ normalized and averaged over 100

randomizations of the original network. Weights have been discretized rounded off to the nearest integer. The inset shows the degree-based rich-club $\rho(k_T)$ of the unweighted representation. Right: a sketch showing greater detail of how the hubs interact among themselves. Darker colors represent inner subsets in the nested hierarchy as defined by the threshold strength. Within a plot, the sizes of the nodes are proportional to their strengths. The numerical values labeling the links represent the ratio of the actual weight of the tie to its average value in the randomized version. (year 2000, DBII in Section I of Appendix).

The analysis of the WTW for the year 2000 (DBII, see Appendix), where the weights give the annual exchanges of merchandizes in millions of current-year U.S. dollars, shows very different behaviors of the weighted and the unweighted rich-club. While the unweighted WTW is dominated by structural connectivity effects, the weighted representation runs at the value 1 just until the tail end, where it exhibits a negative and decreasing strength-based coefficient for large values of the strength, providing evidence of a clear and sharp rich-multipolarisation phenomenon. In this region, a group of rich nodes -the United Kingdom, China, France, Japan, Germany, and the United States, in increasing order of strength- form subsets with negative weighted rich-club coefficient. Notice the overlap of five members —all except China—with the club of the seven largest industrialised and richest countries in the world, the G7, which also includes Italy and Canada (the next two countries in the strength hierarchy of the WTW in 2000). The decreasing tail indicates that these hub countries share on average less weight among them than expected in the random situation. So they seem to have an aversion to connecting to each other, as anticipated between powers in a strongly competitive economic system. However, they interact anyway, forced by structural constraints, and form fully connected subsets.

The average tendency in the interactions between rich countries to loosely interconnect in terms of weight as compared with the random null model counterpart hides local regularities in contrast to the overall behavior. A direct inspection of the subgraphs formed by the hubs, comparing the actual weight in each link with the reference value given by the null model, shows that the biggest world economies seem to be polarized into two connected blocks in direct competition: the United States and its Asian allies Japan and China, against Europe -France, Germany, and the United Kingdom. Each block is tightly connected, shown by trading volumes larger than random predictions, and competition between the two is exposed by the reduced exchange of merchandise as compared to the null model.

Other studies of the rich-club ordering of the WTW in a weighted representation implement different normalization null models. One possibility is to compare the real network with a maximally random weighted network where degrees and strengths are both preserved while the structure is otherwise completely reshuffled. Since the WTW suffers from strong structural constraints, the randomization of the unweighted topology previous to the randomization of weights does not introduce noticeable differences with the previous results.

3. Directed Representations

In the WTW, as in many other systems, interactions between pairs of nodes are asymmetric. This is true both at the topological level (the fact that country i imports from country j does not necessary imply that country j imports from country i) and at the weighted level (even if trade flows in both directions are present, the volumes of the two flows may differ significantly). The undirected network representation becomes then a first order approximation that can be refined by representing the connections as arrows, indicating the country that acts as a source in the trade exchange at the tail and the destination country at the head. Directed network representations are more complete and convey more information about the system, but this comes to the price of the need of a larger number of variables to characterize the network. Nodes' connectivities, for instance, are described by two coexisting degrees k^{in} and k^{out} representing the number of incoming neighbors pointing to it and the number of outgoing neighbors pointed by it respectively, which sum up to the total degree k^{T}. Hence, the degree distribution for a directed network is a joint degree distribution $P(k^{\text{in}}, k^{\text{out}})$ of in- and out-degrees, which in general may be correlated. Moreover, if the directed network is also weighted the total strength s^{T} associated to a certain node has two contributions coming from the incoming strength s^{in} and the outgoing strength s^{out}, which are obtained by summing up all the weights of the incoming or outgoing links respectively.

Notice that different weighted directed reconstructions of the international trade system are possible, depending on whether purely directed links as trade net imbalances between pairs of countries or bidirectional connections with different weights running in each directions are considered, for instance. Moreover, the directionality of all the arrows can be reversed depending on the emphasis put on the flow of merchandizes or on the resulting flow of money. In what follows, we first describe the directed description of the WTW at both the unweighted and weighted levels. Next, we discuss some results about a particular weighted directed version of the WTW, where weights represent net trade imbalances between pairs of countries, different from zero when bilateral imports do not balance exports, and running in the direction of the money flow.

3.1. Directedness and Reciprocity

Taking into account the directionality of trade relationships clearly results in a gain of information. We now describe the effect of introducing the directed description at a purely topological, unweighted level. As we discuss below, at this level the directionality of trade can be simply taken care of in terms of a single overall parameter. This simplifies the directed description considerably. However, as we show later on, this is not the case when the volumes of trade are also considered, as relations that are symmetric at a topological level may be strongly asymmetric at the weighted level.

Let a_{ij} be the adjacency matrix of a directed graph, and b_{ij} the adjacency matrix of the same graph when regarded as undirected. It is easy to show that the two matrices are related via the equation $b_{ij} = a_{ij} + a_{ji} - a_{ij}a_{ji}$. Correspondingly, the directed degrees $k_i^{in} = \sum_j a_{ij}, k_i^{out} = \sum_j a_{ji}$ and the undirected degree $k_i = \sum_j b_{ij}$ computed on the two versions of the graph are related through the following equation:

$$k_i = k_i^{in} + k_i^{out} - k_i^{\leftrightarrow}$$
(7)

where we have introduced the *reciprocal degree* $k_i^{\leftrightarrow} = \sum_j a_{ij}a_{ji}$, defined as the number of neighbors of i with double (reciprocal) connections to and from i. Interestingly, for the specific case of the WTW these quantities are empirically found to obey two important regularities that allow us to recover the main directed features from the undirected ones. Firstly, one always finds empirically that on average $k_i^{\text{in}}(t) \approx k_i^{\text{out}}(t)$ for all i,t (see however Section 3.3 for more resolved analyses). This means that the numbers of exporters and importers of a given country are in general very similar. Secondly, the following linear relation is observed between $k_i^{\leftrightarrow}(t)$ and the total degree $k_i^{\text{T}}(t)$, defined as $k_i^{\text{in}}(t) + k_i^{\text{out}}(t)$:

$$k_i^{\leftrightarrow}(t) \approx r(t)k_i^T(t)/2$$
(8)

Note that the total degree $k_i^{\text{T}}(t)$ is in general different from the undirected degree k_i, their difference being exactly $k_i^{\leftrightarrow}$. The above relation is shown in Figure 8 for some snapshots of the WTW. It reveals that the number of trade partners acting simultaneously as importers and exporters of a given country is proportional to the total number of importers and exporters (counted twice when both roles apply) for the same country.

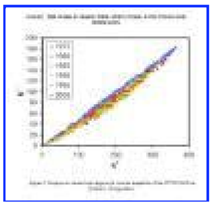

Figure 8. Reciprocal versus total degree for various snapshots of the WTW (DBII in Section I of Appendix).

Note that in all snapshots the proportionality holds, but the coefficient of this proportionality changes with time. Importantly, the factor $r(t)$ can be shown to coincide with the *reciprocity*, defined as the ratio of the number $L^{\leftrightarrow}(t)$ of links pointing in both directions to the total number of links $L(t)$:

$$r(t) = L^{\leftrightarrow}(t)/L(t)$$
(9)

The reciprocity equals zero if no link is reciprocated, and one if all links are reciprocated. The above empirical results allow us to obtain many properties of the WTW, viewed as a directed graph, from its undirected version. For instance, one can obtain

$$k_i^{\text{in}}(t) \approx k_i^{\text{out}}(t) \approx k_i^{\text{T}}(t)/2 \approx k_i(t)/\left[2 - r(t)\right]$$
(10)

and a relation between L and the number L^u of links in the undirected network:

$$L(t) = \sum_i k_i^{\text{in}}(t) = \sum_i k_i(t)/\left[2 - r(t)\right] = 2L^{\text{u}}(t)/\left[2 - r(t)\right]$$
(11)

The above formulas show that in the specific case of the WTW it is possible to recover, at any time step t, many of its fully directed properties from their undirected counterparts through the value of $r(t)$ alone. This is not true in general for other networks, for which no clear relation may exist between the two representations, so that using the undirected version instead of the fully directed description may result in an uncontrolled loss of information. Also note that higher values of reciprocity imply a smaller information loss, as for a perfectly reciprocal network the directed description can be recovered exactly by replacing each link with a pair of reciprocal ones.

It is therefore important to study the value of reciprocity; or rather its evolution as it depends on the particular snapshot considered. While it is of course possible to measure $r(t)$ for various years, this quantity does not actually allow us to compare different snapshots of the same network consistently. This is because networks with different numbers of links have different expected reciprocities even in the uncorrelated case, and thus both the numerator and the denominator of $r(t)$ change with time, without a clear mutual relation. An unbiased estimator of reciprocity is the quantity

$$\rho(t) = \frac{r(t) - r^{\text{ran}}(t)}{1 - r^{\text{ran}}(t)}$$
(12)

measuring the reciprocity with respect to the random expectation r^{ran}, which is simply equal to the density of links $L/N(N-1)$. The above quantity can be simply derived as a correlation coefficient between reciprocal entries of the adjacency matrix, and ranges between -1 and 1. For a perfectly reciprocal network one has $\rho(t) = 1$, while for a network with no tendency towards the formation of mutual links (other than by chance) $\rho(t) = 0$. In general, a positive value of $\rho(t)$ indicates that reciprocal links tend to be favored, while a negative value indicates that reciprocal links tend to be avoided. This allows a consistent comparison across different years, and more in general across networks with different numbers of vertices and links. The evolution of both $r(t)$ and $\rho(t)$ is shown in Figure 9. It can be seen that both measures fluctuate about an approximately constant value up to the early 1980’s, and then increase steadily. However, the evolution of $\rho(t)$ highlights that, once the changes in the density of the network are controlled for, the increasing trend of the reciprocity is steeper than indicated by $r(t)$. In particular, the international trade system appears to have undergone a rapid reciprocation process starting from the 1980’s. It is instructive to combine this result with the previously mentioned constant trend of the link density in the undirected version of the WTW. As at the undirected level there is no increase of link density, the rapid increase of reciprocity signals many new directed links being placed between countries that had already been trading in the opposite direction, rather than new pairs of reciprocal links being placed between previously non-interacting countries. In other words, many pairs of countries that had previously been trading only in a single direction have been establishing also a reverse trade channel, and this effect dominates on the formation of new bidirectional relationships between previously non-trading countries.

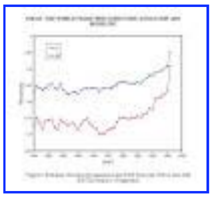

Figure 9. Evolution of reciprocity measures in the WTW from year 1950 to year 2000 (DBII in Section I of Appendix).

The results reported so far imply that, at the unweighted level, the directedness of trade channels can be in a good measure recovered from the undirected description in terms of the reciprocity parameter. Moreover, the reciprocity appears to be evolving to values so high that most links can be considered bidirectional, justifying an undirected approach. We now consider the corresponding problem in the weighted case. A measure of the asymmetry of a directed weighted network has been recently proposed as the distance between the weighted adjacency matrix $\mathbf{W}$ of the network and its transpose $\mathbf{W}^{\mathrm{T}}$ as follows:

$$S(\mathbf{W}) = \left(\frac{N+1}{N-1}\right)\frac{\left\|\mathbf{W}-\mathbf{W}^{\mathrm{T}}\right\|_{\mathrm{F}}^{2}}{\left\|\mathbf{W}\right\|_{\mathrm{F}}^{2}+\left\|\mathbf{W}^{\mathrm{T}}\right\|_{\mathrm{F}}^{2}}$$

(13)

where $\|\mathbf{W}\|_{\mathrm{F}}^{2}$ is the square of the so-called Frobenius (or Hilbert-Schmidt) norm of the matrix $\mathbf{W}$, defined as

$$\|\mathbf{W}\|_{\mathrm{F}}^{2} = \|\mathbf{W}^{\mathrm{T}}\|_{\mathrm{F}}^{2} = \sum_{i,j=1}^{N} w_{ij}^{2}$$

(14)

Using this index, it is possible to show that the weighted WTW is strongly symmetric, with pairs of reciprocal links carrying weights of similar magnitude. This result justifies the statistical analysis of the weighted WTW as an undirected network, and leads to the approach described in Section 2.2 where weights are symmetrised. The resulting study has already been described in that section.

However, even if the volumes of trades running in opposite directions are statistically similar, this does not mean that the net flow between pairs of countries is exactly zero. If this were the case, net exports of all countries would vanish and have no effect on the Gross Domestic Product. Moreover, trades pointing only in one direction are not captured by the symmetrised representation. It is therefore important to add one more level of description and focus on net trade flows defined as the difference between trade volumes running in opposite directions. These flows represent trade imbalances and offer an intrinsically directed description, where any two countries are connected by at most one link transferring a net amount of money in a specific direction. This means that there are no reciprocated links, i.e. $L^{\leftrightarrow} = 0$, and therefore the reciprocity parameters defined above are $r = 0$ and $\rho = -r^{\mathrm{ran}}/(1-r^{\mathrm{ran}})$. The analysis of this representation of the WTW is described in the following sections.

1. Introduction

3.2. Time Invariance of the Distribution of Bilateral Trade Imbalances

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THE WORLD TRADE WEB: STRUCTURE, EVOLUTION AND MODELING

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Contents

3.2. Time Invariance of the Distribution of Bilateral Trade Imbalances

The evolution in time of the distribution of bilateral trade flows shows a striking phase transition. Until 1960, the distributions for every year overlapped in a characteristic function, which afterwards evolved widening as time goes by. Most interestingly, the different distributions of trade imbalances for all years since 1960 can be rescaled into a single master curve just by taking into account the evolution in time of the global GDP, which marks a characteristic scale with respect to which the system is self-similar. The breaking of the original folding at the point when a new single-curve collapse suggests that the rule that has been governing the statistical behavior of bilateral trade imbalances until the 1960s has changed to a new scaling law in the last decades, and that this has happened in a sudden transition of the world trade system just at the beginning of the last wave of globalization.

As the top plot in Figure 10 shows, the time evolution of global GDP seem to be decoupled until the 1960s to that of the average bilateral trade flow. From 1870 to that date, the average imbalance fluctuated around a constant level, in contrast to the estimation for the global GDP, which grew appreciably. Afterwards, they coupled and follow similar growth patterns. This seems indicative of a change of behavior or transition. In Figure 10 (middle plot), we present the complementary cumulative probability distribution of net trade flows between pairs of countries for several different years since 1870. The curves are measuring the probability that a trade imbalance between two countries in the trade system is bigger than a certain amount, and the cumulative evaluation offers the advantage of filtering out the statistical noise due to the finite sizes of the samples without losing information about the distribution.

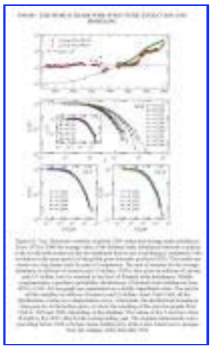

Figure 10. Top: Historical evolution of global GDP values and average trade imbalances. From 1870 to 2000 the average value of the bilateral trade imbalances between countries in the world trade system for the two databases that we are considering is compared to the evolution in the same period of the global gross domestic product (GDP). The results are shown on a log-linear scale for ease of comparison. The unit of measure for the average imbalance is millions of current year US dollars. GDP is also given in millions of current year US dollars, but it is rescaled to the level of bilateral trade imbalances. Middle: complementary cumulative probability distributions of bilateral trade imbalances from 1870 to 1990. All the graphs are represented on a double logarithmic scale. The unit for all the variables is millions of current year US dollars. Inset: Until 1960, all the distributions overlap on a characteristic curve. Afterwards, the distributions broaden as time goes on. In the bottom plots, we show the rescaling of the previous graphs from 1960 to 1990 and 2000, depending on the database. The values on the F axis have been divided by the GDP value for the corresponding year. The original characteristic curve prevailing before 1960 is broken (main middle plot), while a new master curve emerges from the collapse of the data after 1960.

To a good approximation, all the curves overlap between the years 1870 and 1960 (see the inset of Figure 10), moment in which the folding is broken and the distributions evolve, widening year after year. The curve for each year can be rescaled by taking into account the global GDP value in that period.

For each distribution, the transformation $F \to F/GDP$, which divides the fluxes in the horizontal F axis by the corresponding global GDP value, is applied. Notice that this transformation would produce the same result if real values were used instead of nominal values. The results are shown in the bottom graphs of Figure 10. All the distributions for different years from 1960 until 2000 and for the two different databases under consideration show an excellent collapse into a single master curve. Imports and exports on their own are seen to present the same behavior, due to the high correlation between the levels of imports and exports in every single trade channel. All these distributions are log-normal—they can be thought of as the multiplicative product of many small independent factors—again a ubiquitous shape in economics. They can be adjusted to the form $1/2 - 1/2\,\mathrm{erf}\left[\left(\ln(x)-\mu\right)/\left(\sigma\sqrt{2}\right)\right]$, where μ and σ are the mean and standard deviation of ln(x) and erf stands for the error function. For the master curve in the left bottom plot the parameters are $\mu = -14.10$ and $\sigma = 2.34$, and for the right bottom plot $\mu = -14.39$ and $\sigma = 2.55$, so the two databases produce consistent information. The lognormal shape is not only found for the distribution of imbalances but also for the distribution of link weights measured as the total trade volume between pairs of countries.

The collapse of the distribution for the different years into a master curve implies that the system is self-similar with respect to the characteristic scale given by the GDP. That means that the widening of the distributions in time is just a dilatation driven by the increase in total GDP, and the same curve is found once the growth in GDP is discounted. Until new structural changes impact upon the world trade system, we can assume that this behavior will be preserved through time so that, by taking into account global GDP projection values, we can predict the statistical distribution of bilateral trade imbalances that would correspond to future periods.

3.3. Network of Bilateral Trade Imbalances

Flux analysis in the international trade system can be tackled at a global scale. In the weighted directed network of merchandize trade imbalances between world countries, we define the elements of the weighted adjacency matrix **F** as $F_{ij} = |T_{ij}| = |T_{ji}|$ for all i, j with $T_{ij} < 0$, and $F_{ij} = 0$ for all i, j with $T_{ij} \geq 0$, where $T_{ij} = E_{ij} - I_{ij}$, is the net money flow from country i to country j due to trade exchanges and the elements E_{ij} measure the exports of country i to country j while I_{ij} measure the imports of country i from country j . With this procedure, yearly snapshots in the period 1948-2000 characterizing the time evolution of the trade network have been reconstructed from data reported in DBII, see Appendix 1.

From **F**, the net trade imbalance of a country j can be computed as $\Delta s_j = s_j^{\text{in}} - s_j^{\text{out}}$, where the incoming and outgoing strengths are computed as

$s_j^{\text{in}} = \sum_i F_{ij}$ and $s_j^{\text{out}} = \sum_i F_{ji}$ respectively. Depending on Δs_j, countries can be defined as net consumers or net producers. Net producers export more than they import, the total outcome being a trade surplus which corresponds to a positive net trade imbalance, whereas net consumers export less than they import, the total outcome being a trade deficit, which is indicated by a negative net trade imbalance. Since one incoming link for a given country is always an outgoing link for another, the sum of all the countries' trade imbalances in the network must be zero. While the local balance is not conserved, the complete system is a closed system globally balanced with conservation of the total flux. Merchandize, or equivalently money, flows in the system from country to country, with the peculiarity that there is a global current of money from consumer countries to producer ones.

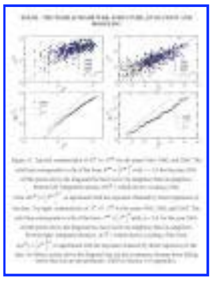

Figure 11. Top left: scattered plot of k^{in} vs. k^{out} for the years 1960, 1980, and 2000. The solid line corresponds to a fit of the form $k^{\text{out}} \propto \left[k^{\text{in}}\right]^{\nu}$ with $\nu = 0.5$ for the year 2000. All the points above the diagonal line have more out-neighbors than in-neighbors. Bottom left: integrated measure $A(k^{\text{in}})$, which shows a scaling of the form $A(k^{\text{in}}) \propto \left[k^{\text{in}}\right]^{1.5}$, in agreement with the exponent obtained by direct regression of the data. Top right: scattered plot of s^{in} vs. s^{out} for the years 1960, 1980, and 2000. The solid line corresponds to a fit of the form $s^{\text{out}} \propto \left[s^{\text{in}}\right]^{\mu}$ with $\mu = 0.6$ for the year 2000. All the points above the diagonal line have more out-neighbors than in-neighbors. Bottom right: integrated measure $A(s^{\text{in}})$, which shows a scaling of the form $A(s^{\text{in}}) \propto \left[s^{\text{in}}\right]^{1.6}$, in agreement with the exponent obtained by direct regression of the data. As before, points above the diagonal line are net consumers whereas those falling below this line are net producers. (DBII in Section I of Appendix).

In this network, the number of incoming and the number of outgoing connections of a given country are positively correlated quantities. This means that countries with high in degree have also high out degree. The same happens with the incoming and outgoing strengths, so that the highest the total incoming flux the highest the total outgoing flux. The scattered plots of k^{out} versus k^{in} and s^{out} versus s^{in} for the years 1960, 1980 and 2000 are shown in Figure 11. The positive correlation is evident but it is made clearer by the integrated quantities $A(k^{\text{in}}) = \int^{k^{\text{in}}} k^{\text{out}}(k'^{\text{in}})dk'^{\text{in}}$ and $A(s^{\text{in}}) = \int^{s^{\text{in}}} s^{\text{out}}(s'^{\text{in}})ds'^{\text{in}}$ in order to smooth out noise (correspond to the areas under the signals shown on the top graphs of Figure 11). If the relation between the in and out quantities is linear, the integrated measure scales quadratically, whereas it scales with an exponent in the range (1,2) if the relation is sub-linear. The bottom graphs of Figure 11 show the functions $A(k^{\text{in}})$ and $A(s^{\text{in}})$, which exhibit a clear scaling with exponents 1.5 and 1.6, in perfect agreement with the results obtained by a direct regression of the data. This sub-linear behavior implies that countries with small s^{in} have small s^{out}, but for them the value of s^{out} is on average bigger than that of s^{in} (points are above the diagonal in the small values region of the right top graph of Figure 11). At the same time, countries with high s^{in} have high s^{out}, but for them the value of s^{out} is on average smaller than that of s^{in} (points are below the diagonal in the large values region of the right top graph of Figure 11). The same can be said from the results for the degrees k^{in} and k^{out} in the top graph of Figure 11. All this indicates that small economies tend to be net consumers, hence with $\Delta s < 0$, whereas big ones tend to be net producers, so with $\Delta s > 0$.

2.2. Weighted Structure 3.3.1. The Backbone of the WTW

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3.3.1. The Backbone of the WTW

The imbalance networks show a high density of connections and heterogeneity of the fluxes among countries. Indeed, as the number of countries increases, so does the average number of trade partners, as well as the total flux of the system, which is seen to grow proportional to the aggregated world Gross Domestic Product (see Section 3.2). The overall flux organization at the global scale can be characterized by the study of the flux distribution. A first indicator of the system heterogeneity is provided by the probability distribution $P(F_{ij})$ denoting the probability that any given link is carrying a flux F_{ij}. The observed distribution is heavy-tailed and spans approximately four orders of magnitude (see Section 3.2). Such a feature implies that only a small percentage of all the connections in the network carry most of its total flow F and that there is no characteristic flux in the system, with most of the fluxes below the average and some of them with a much higher value. This is however not totally unexpected since a large scale heterogeneity is a typical feature of large-scale networks. In addition, the global heterogeneity could just be due to differences in the sizes of the countries, in their population and in their respective Gross Domestic Product. More interesting is therefore the characterization of the local heterogeneity; i.e., given all the connections associated to each given country, how is the flux distribution for each of them.

Local heterogeneity implies that only a few links carry the biggest proportion of the country's total in-flow or out-flow. Interestingly, such heterogeneity would define specific pathways within the network that accumulate most of the total flux. Heterogeneities at the local level can be detected by the disparity function, that for each country i with k incoming or outgoing trade partners is given by $kY_i(k) = k\sum_{j=1}^{k} p_{ij}^2$, where k can be either k^{in} or k^{out} in order to discern between heterogeneities in incoming and outgoing fluxes separately, and where the normalized fluxes of node i with its neighbors are calculated as $p_{ij} = F_{ji} / s_i^{\text{in}}$ for incoming connections and as $p_{ij} = F_{ij} / s_i^{\text{out}}$ for the outgoing ones. The function $Y_i(k)$ is extensively used in economics as a standard indicator of market concentration, referred to as the Herfindahl–Hirschman Index or HHI, and it was also introduced in the complex networks literature as the disparity measure. In all cases, $Y_i(k)$ characterizes the level of local heterogeneity. If all fluxes emanating from or arriving to a certain country are of the same magnitude, $kY_i(k)$ scales as 1 independently of k, whereas this quantity depends linearly on k if the local flux is heterogeneously organized with a few main directions. Increasing deviations from the constant behavior are therefore indicating heterogeneous situations in which fluxes leaving or entering each country are progressively peaked on a small number of links with the remaining connections carrying just a small fraction of the total trade flow. On the other hand, deviations from the constant behavior have to be expected for low values of k and it is important to compare the obtained results with the deviations simply produced by statistical fluctuations.

It is necessary then to introduce a null model for the distribution of flows among a given number of neighbors in order to assess, in a case by case basis, whether the observed heterogeneity can just be due to fluctuations or it is really significant (see Appendix 2). In Figure 12, we show the empirical measures along with the region defined by the average value of the same quantity $kY_i(k)$ plus two standard deviations as given by the null model (shadowed area in brown). For a homogeneously random assignment of weights, this quantity converges to a constant value for large k, which is clearly different from the observed empirical behavior. Most empirical values lie out of the null model domain, which proves that the observed heterogeneity is due to a well definite ordering principle and not to random fluctuations. The direct fit of the data indicates that both in and out fluxes follow the scaling law $kY(k) \propto [k]^{\beta}$ with $\beta_{\text{in}} = 0.6$ for the incoming connections and $\beta_{\text{out}} = 0.5$ for the outgoing ones (see Figure 12). This scaling represents and intermediate behavior between the two extreme cases of perfect homogeneity or heterogeneity but clearly points out to the existence of strong local heterogeneities. The emerging picture is therefore consistent with the existence of major pathways of trade flux imbalances (thus money) that enters the country using its major incoming links and leaves it through its most inhomogeneous outgoing trade channels.

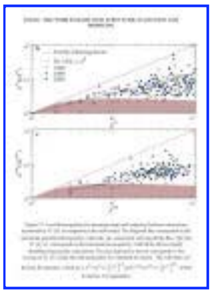

Figure 12. Local heterogeneity for incoming (top) and outgoing (bottom) connections measured by $kY(k)$ as compared to the null model. The diagonal line corresponds to the maximum possible heterogeneity, with only one connection carrying all the flux. The line $kY(k) = 1$ corresponds to the maximum homogeneity, with all the fluxes equally distributed among the connections. The area depicted in brown corresponds to the average of $kY(k)$ under the null model plus two standard deviations. The solid lines are the best fit estimates, which give $k^{\text{in}}Y(k^{\text{in}}) \propto \left[k^{\text{in}}\right]^{0.6}$ and $k^{\text{out}}Y(k^{\text{out}}) \propto \left[k^{\text{out}}\right]^{0.5}$. (DBII in Section I of Appendix).

The analysis of the local heterogeneities in the trade fluxes prompts to the presence of high-flux backbones, sparse subnetworks of connected trade fluxes carrying most of the total flux in the network. This backbone is necessarily encoding a wealth of information being the dominating structure of the trade system. It is also worth remarking that the local heterogeneity is not just encoded in high flux links in terms of their absolute intensities, but also takes into account the local heterogeneity by comparing the strength of the fluxes associated to a given country with its total strength. It is then interesting to filter out this special links and provide snapshots of the trade system backbone. This can be achieved by comparing the link fluxes with the null model used for the calculation of the disparity in a pure random case. The same null model allows us to calculate for each connection of a country i the probability α_{ij} that its normalized flux value p_{ij} is due to chance. Along these lines, we can identify highly inhomogeneous fluxes as those that satisfy

$$\alpha_{ij} = 1-(k-1)\int_0^{p_{ij}} (1-x)^{k-2}\,dx < \alpha \qquad (15)$$

where α is a fixed significance level. Notice that this expression depends on the number of connections of each country, k. Choosing a global threshold for all countries gives a homogeneous criteria that enables to compare heterogeneities in countries with different number of connections and filter out links that carry fluxes which can be considered not compatible with a random distribution with an increasing statistical confidence. The backbone is then obtained by preserving all the links that beat the threshold for at least one of the two countries at the ends of the link while discounting the rest. By changing the significance level we can filter out the links focusing on progressively more relevant heterogeneities and backbones. An important aspect of this filtering algorithm is that it does not belittle small countries and then, it offers a systematic procedure to reduce the number of connections without diminishing the number of countries and choosing the backbone according to the amount of trade flow we intend to characterize. It provides a quantitative and consistent way to progressively identify the relevant flow backbone once the level of statistical confidence with respect to the null case is fixed or, instead, the total flow to be represented in the system. Indeed, it is remarkable that when looking at the network of the year 2000 one finds that the $\alpha = 0.05$ backbone contains only 15% of the original links yet accounting for 84% of the total trade imbalance. Most of the backbones form a giant connected component containing most of the countries in the network, and only for very high values of the confidence level, defining a sort of *super-backbones*, some disconnected components appear and the number of countries starts to drop. In this respect, the $\alpha = 0.01$ backbone seems to offer the best trade-off since it keeps nearly all countries, 75% of the total trade imbalances, and one order of magnitude less connections than in the original network (see Table 1). The backbone reduction is extremely effective in sorting out the most relevant part of the network and can be conveniently used for visualization purposes. For the sake of space and reproduction clarity, we report the backbones corresponding to $\alpha = 10^{-3}$, still accounting for approximately 50% of the total flux of the system. Figure 13 shows two snapshots of such backbones for 1960 and 2000. These high-flux backbones evidence geographical, political and historical relationships among countries, which affect the observed trade patterns. For instance, the trade of US with its geographically closer neighbors and also continental neighbors, the case of Russia and the former Soviet republics, or France and its former colonies, the lack of strong trade relations between the two blocks in the cold war, more evident in 1960. In general terms, a recurrent motif present in all years is the star-like structure, formed by a central powerful economy surrounded by small dependent economies. The USA appears as one of those powerful hubs during all this period. However, other countries have gradually lost this role in favor of others. This is the case of the UK, which was the only star-like counterpart of the USA in 1948; since then its direct area of influence has been narrowing. On the contrary, other countries have arisen for different reasons as new hub economies. This is the case of some European countries, Japan, and most recently, China. It is important to notice that all the highlighted effects are uncovered by the filtering of the data without any previous assumption or annotated analysis on the relationships between countries.

Table 1. Percentage of the original total weight F, number of nodes N and links E in the 1960 and 2000 imbalance networks that remain in the backbone as a function of the significance level α. Reproduced from Serrano et al. (2007).

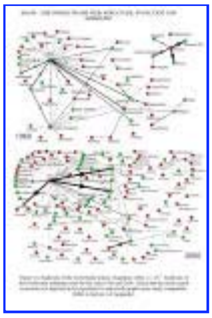

Figure 13: Backbone of the world trade system. Snapshots of the $\alpha = 10^{-3}$ backbone of the world trade imbalance web for the years 1960 and 2000. Notice that the most central economies are depicted at fixed positions to make both graphs more easily comparable. (DBII in Section I of Appendix).

3.3.2. Diffusion on the WTW and the Dollar Experiment

The WTW is a directed flow network that shares similarities with other networks in this category, such as metabolic networks that transfer and process energy and matter or technological systems that transport information. Indeed, the trade imbalances network transports money (in a first approach, debt is not taken into account). In other words, net consumer countries are injecting money in the system, and money flows along the edges of the network to finally reach producer countries. Producer countries, however, do not absorb completely the incoming flux, redistributing part of it through their outgoing links. The network is therefore characterizing a complex dynamical process in which the net balance of incoming and outgoing money is the outcome of a global diffusion process. The realization of such a non-local dynamics in the flow of money due to trade imbalances spurs the issue of what impact this feature might have on the effect that one economy can have on another. In order to tackle this issue, it is informative to perform a simple numerical study, defined as the "*dollar experiment*".

The experiment considers running on the networks two symmetric random walk processes. Since empirical data are limited by yearly frequency, the first approximation is that the time scale of the changes in the structure of the underlying trade imbalances network is bigger than the characteristic diffusion time of the random walk processes. In the first random walk process, a consumer country with negative net trade imbalance injects one dollar from its net debit into the system. The dollar travels through the network following fluxes chosen with a probability proportional to their intensity, and has as well a certain probability of being trapped in producer countries (positive net trade imbalance) with a probability $P_{\text{abs}} = \Delta s / s^{\text{in}}$. More precisely, a consumer country, such as the USA, puts one dollar into the system that goes from country to country always following outgoing fluxes chosen with a probability proportional to their intensity. If in its way it finds another source it just crosses it, whereas if it finds a producer country j it has a probability $P_{\text{abs}}(j)$ of being absorbed. Mathematically, this process is a random walk on a directed network with heterogeneous diffusion probability and in the presence of sinks. By repeating this process many times it is possible to obtain the probability e_{ij} that the traveling dollar originated in the source i is finally absorbed in the sink j. In other words, for each dollar that a source country i adds to the system, e_{ij} represents the fraction of that dollar that is retained in country j. The second symmetric process considers that each producer country is receiving a dollar and the traveler dollar goes from country to country always following incoming links backward chosen with a probability proportional to their intensity. If in its way it finds another sink it just crosses it, whereas if it finds a source j it has a probability $P_{\text{abs}} = |\Delta s| / s^{\text{out}}$ of remaining in that country. The iteration of this process gives the probability g_{ij} that yields the fraction originated in the source country j of each dollar that a sink country retains. The matrices e_{ij} and g_{ij} are normalized probability distributions and are related by the detailed balance condition $|\Delta s_i| e_{ij} = \Delta s_j g_{ji}$. Using this property we can write $\Delta s_j = \sum_{i:\text{source}} e_{ij} |\Delta s_i|$ and $\Delta s_i = \sum_{j:\text{sink}} g_{ji} |\Delta s_j|$, which means that the total trade imbalance of a sink or source country can be written as a linear combination of the trade imbalances of the rest of the source or sink countries, respectively. Therefore, by measuring e_{ij}, it is possible to discriminate the effect that one economy has on another or, with g_{ij}, to find out which consumer country is contributing the most to a producer one, in both cases taking into account the whole topology of the network and the heterogeneities of the fluxes. The advantage of this approach lies on its simplicity and the lack of tunable parameters. Indeed, all the information is contained in the network itself, without assuming any kind of modeling on the influences among countries.

Table 2. Top: effect of two major source countries, USA and Switzerland, on the rest of the world. The first list is a top ten ranking of countries according to e_{ij}, where the index i stands for the analyzed source. The second list is the top ten ranking of direct bilateral trade measured as the percentage of flux from the source country, that is, $e_{ij}^{\text{local}} = F_{ij} / s_i^{\text{out}}$. Bottom: major contributors to two major sink countries, Japan and Russia. The first list is a top ten ranking of countries according to g_{ij}, i standing for the analyzed sink. The second list is the top ten ranking due to direct trade. In this case, the direct

contribution is $g_{ij}^{\text{local}} = F_{ji} / s_i^{\text{in}}$. Countries in boldface have no direct connection with the analyzed country. The values for e_{ij} and g_{ij} are obtained from the simulation of the dollar experiment described in the text using 10^6 different realizations for each country, for the year 2000. Reproduced from Serrano et al. (2007).

By using this experiment it is possible to evaluate for a consumer country where the money spent is finally going. For each dollar spent we know which percentage is going to any other producer country and we can rank those accordingly. It is important to remark that in this case countries might not be directly connected since the money flows along all possible paths, sometimes through intermediate countries. This kind of ranking is therefore different from the customarily considered list of the first neighbors ranked by magnitude of fluxes. The analysis indeed shows unexpected results and, as it has been already pointed out in other works applying other methodologies, a country can have a large impact on other countries despite being a minor or indirect trading partner, see Table 2. Similarly, producer countries may have a share of the expenditure of non-directly connected countries resulting in a very different ranking of their creditors. As an example, for each net dollar that the USA injects into the system, only 9.3% is retained in China although the direct connection imbalance between these countries is 16.7%. Very interestingly, we find that Switzerland spent a large share of his trade imbalance in countries that do not have appreciable trade with it and are, therefore, not directly connected such as Japan, Indonesia, and Malaysia. The Swiss dollars go to these countries after a long path of trade exchanges mediated by other countries. By focusing on producer countries we find other striking evidence. While the first importer from Russia by looking locally at the ranking of trade imbalances is Germany, the global analysis shows that one third of all the money Russia gains from trade is coming directly or indirectly from the USA. In Table 2, we report other interesting anomalies detected by the global analysis.

4. Modeling the World Trade Web (WTW)

In this section we review some theoretical approaches to the problem of modeling the WTW. In some sense, the earliest and most traditional approach is that of gravity models, which belong to a class of models widely explored in the literature on economic and transport systems. These models aim at explaining the observed magnitude of trade (or traffic) fluxes, but fail to explain their topology. By contrast, more recent network-inspired approaches succeed in reproducing the topology of trade, but do not make predictions about link weights. So at present it appears that a combination of both approaches is necessary in order to understand the WTW structure at various levels of organization.

4.1. Gravity Models

Attempts to estimate the magnitude of international trade flows using the so-called “gravity equations” (termed so due to an analogy with Newton’s law of gravitation) date back at least to the pioneering approach of Tinbergen. Since then, various alternative forms of the gravity equations have been proposed. The basic idea is that the volume of trade between two countries is positively related to some proxy of economic size of both countries, and is negatively related to some measures of “trade resistance” between them. It is expected that the dominant factor determining resistance to trade is geographic distance. Other factors, either favoring or suppressing trade, can however be included using a straightforward procedure. For instance, if both geographical distance and another factor are considered, a simple form of the gravity equations in international trade becomes

$$w_{ij} = \beta_0 \left(w_i\right)^{\beta_1} \left(w_j\right)^{\beta_2} \left(d_{ij}\right)^{\beta_3} \left(f_{ij}\right)^{\beta_4} \qquad (16)$$

where w_{ij} is the expected trade flow from country i to country j in a given year, w_i and w_j are the nominal GDPs (in the same year) of country i and j respectively, d_{ij} is the distance from the economic center (main economic location) of country i to that of country j, f_{ij} is another factor either favoring or suppressing trade between i and j. The parameters $\{\beta_0, \ldots, \beta_4\}$ tune the effect of each factor on the magnitude of trade flows, and can be used to fit the gravity model to real data. Importantly, these parameters are global (i.e. each parameter has the same value for all pairs of countries at a given time), and therefore the model seeks to explain the observed heterogeneity of trade flows in terms of a few country-dependent quantities (mainly, GDP and geographic position). For this reason, usually an error term is added to the above equation, as it is clearly impossible that a single set of parameters matches trade volumes among all pairs of world countries simultaneously. Thus gravity models provide expected pairwise trade volumes, and deviations from these expected values are interpreted as statistical fluctuations. This is in accordance with the methods of parameter estimation most frequently used for gravity models, such as least squares fit.

The expected positive effect of a country’s economic size on its trade relationships implies positive expected values of β_1 and β_2. Indeed, in empirical studies they are both found to be positive, generally similar (indicating some level of reciprocity between mutual trade flows) and not distant from one. By contrast, the expected negative effect of geographic distance on trade implies a negative expected value of β_3, which is again confirmed by empirical studies. The analogy with gravitational force (or energy) would hold for the particular parameter choice $\beta_1 = \beta_2 = 1$ and $\beta_3 = -2$ (or $\beta_3 = -1$) if the GDP is regarded as the “mass” of countries. The value of β_4 depends on whether factor f_{ij} favors or resists trade between i and j. Common choices for f_{ij} or additional factors are membership of the same two countries to a trade agreement or association, or special geographic conditions such as sharing a common border. In such cases, the factor has a positive effect which is confirmed by positive fitted values of β_4 or additional parameters. Opposite negative effects apply for resisting factors such as embargo policies and trade restrictions. Clearly, the number and nature of additional factors included in a gravity model depend on the set of countries considered and the geographic/economic areas they represent.

In general, gravity models reproduce well many datasets of regional and international trade flows. As a consequence, they have dominated the empirical research on trade volumes. However, they do not predict zero trade flows unless in very specialized approaches. This problematic aspect is accompanied by (and related to) the uncertain theoretical underpinnings of gravity models. Although several studies have shown that the form of gravity equations can be derived from theoretical principles, it has also been pointed out that different forms may be obtained under different hypotheses. Therefore it appears that theoretical investigations of gravity models are still needed, which makes them an active field of research.

4.2. Network Models

In network language, gravity models assume that the World Trade Web is a fully connected weighted network, and its topology a complete graph. As we have shown, this is not the case and therefore gravity models fail to predict the rich topology of the international trade network. In what follows, we describe a class of models that reproduce the topology, either directed or undirected, of the World Trade Web. They are a particular case of a type of models, known as “fitness” or “hidden variable” models, that have been proposed in network theory. The success of this approach is an example of how fruitful an interdisciplinary study of the world trade system can be. Interestingly, this class of models retains the basic ingredient of gravity models - i.e. that GDP is the main factor determining trade - but uses this information for the different purpose to derive the expected topology, rather than the expected volumes, of trade interactions.

3.2. Time Invariance of the Distribution of Bilateral Trade Imbalances
4.2.1. Hidden Variable Model

THE WORLD TRADE WEB: STRUCTURE, EVOLUTION AND MODELING

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Keywords: Economy, International Trade System, World Trade Web, Complex Networks.

Contents

4.2.1. Hidden Variable Model

The simplest of all graph models is the Erdos-Renyi one, where it is assumed that an undirected link is placed, independently and with the same probability p, between any pair of vertices. This simple process generates a network with binomial degree distribution, so that the degrees of all vertices are sharply peaked about the expected value $\langle k \rangle = p(N-1)$, where N is the number of vertices. Note that the stochastic nature of the model implies that, in a given realization, the degree of any vertex may be different from its expected value. The homogeneous structure of the model is also manifest in the absence of degree correlations and hierarchical structure. This is in sharp contrast with the observed properties of real networks that we have described.

There is however a simple modification of the Erdos-Renyi model that allows to generate networks with a high degree of heterogeneity. This is the “fitness” or “hidden variable” model, where the heterogeneity is explicitly introduced at the level of vertices. In this model each vertex i is assigned a *fitness* x_i drawn from a given distribution $\rho(x)$. Taken a pair of vertices i and j, a link is drawn between them with fitness-dependent probability $p_{ij} = p(x_i, x_j)$. In other words, the probability that two vertices are connected is assumed to be a function of the *fitness* associated to each of them. As a consequence, all the expected topological properties of the network come to depend only on the functional form of the connection probability $p(x_i, x_j)$ and on the fitness distribution $\rho(x)$. We will consider explicit expressions later on.

While the choice of the ingredients of the model is in principle arbitrary, if one is interested in defining null models of networks with given constraints it is possible to derive the corresponding form of the connection probability uniquely. For instance, if one is interested in generating a maximally random graph with specified average degree, it can be shown that the corresponding choice is the “trivial” one $p(x_i, x_j) = p$. This clearly reduces to the Erdos-Renyi case, where there is no dependence on the form of $\rho(x)$. Since the only constraint is the average degree, all graphs with the same value of this property are generated with equal probability by this model. Technically, the proof that this choice is the maximally random one for graphs with the only constraint on the average degree (or equivalently on the number of links) involves the maximization of the entropy of the ensemble of all possible undirected graphs, subject to the constraint enforced. This formal approach can be generalized easily to other constraints. In particular, maximally random networks where the expected degrees of all vertices (the *degree sequence* $\{k_i\}$) are specified correspond to the choice

$$p(x_i, x_j) = \frac{\delta x_i x_j}{1 + \delta x_i x_j} \qquad (17)$$

In the above expression, δ is a global parameter controlling the expected total number of links, and the $\{x_i\}$ are N parameters that distribute the total number of links among vertices in such a way that each vertex has its specific expected degree. In this model, any two graphs with the same degree sequence are generated with the same probability, in accordance with the requirement that the degree sequence is the only constraint and all other properties are maximally random. Also, any two vertices i and j with the same values $x_i = x_j$ are statistically similar, i.e. they have the same expected degrees. If other constraints were enforced, different forms of the model would be obtained. In general, the class of maximally random networks with fixed constraints has been shown to coincide with the so-called exponential random graphs (also termed p^* or logit models) that have been introduced in sociology as models of social networks.

These considerations have been exploited to suggest the above model as a null model of the WTW. If, similarly to what is assumed in gravity models, one expects that the total GDP of world countries is the main factor determining the topology of trade relationships, then one is led to the expectation that the set of GDP values of all countries determines the corresponding expected degrees. This expectation implies that the probability that two countries are connected in the WTW is given by the above formula, where x_i is a function of the GDP of country i alone. The simplest such function is a linear dependence. Since any coefficient of this proportionality can be reabsorbed in the parameter δ, it is useful to define the adimensional rescaled variable

$$x_i(t) = \frac{w_i(t)}{\sum_{j=1}^{N(t)} w_j(t)/N(t)} \qquad (18)$$

where $w_i(t)$ denotes the total GDP of country i in year t (so that x_i is simply the GDP relative to the average GDP in the same year) and assume the following linkage probability in year t:

$$p_{ij}(t) = \frac{\delta(t)x_i(t)x_j(t)}{1+\delta(t)x_i(t)x_j(t)} \qquad (19)$$

Once the form of $p_{ij}(t)$ is chosen, all the expected topological properties predicted by the model can be compared with the empirical ones. This allows us to check whether the assumptions made, in particular the proportionality between the GDPs and the hidden variables, reproduce the observed topology of the WTW. Note that the only free parameter for a given year t is $\delta(t)$, as the values $\{x_i(t)\}$ are empirically accessible (see Appendix 1 for databases on GDP data). This parameter can be fixed by requiring that the expected number of links at time t

$$\langle L(t)\rangle = \sum_{i<j} p_{ij}(t) \qquad (20)$$

equals the observed one $L(t)$. It is possible to show that this parameter choice maximizes the likelihood to generate the observed network under the model considered. After this unique parameter choice is made, one can use the model to generate random realizations of the WTW topology, and also compute analytically many of the resulting expected (i.e. averaged over many realizations) properties. For instance, the expected degree of vertex i can be obtained as

$$\langle k_i(t)\rangle = \sum_{j\neq i} p_{ij}(t) \tag{21}$$

which is an increasing function of $x_i(t)$, since $p_{ij}(t)$ is an increasing function of both $x_i(t)$ and $x_j(t)$. This simple result already allows one to check whether two experimental facts discussed above are reproduced: one is the empirical form of the degree distribution $P(k)$ shown previously in Figure 2, and the other one is the observed dependency between GDP and degree shown in Figure 3. In Figure 14 we show again the empirical degree distribution (in cumulative form, now from database DBII in Section I of Appendix) for the particular 1995 snapshot, superimposed to the corresponding distribution predicted by the above model. The agreement is very good.

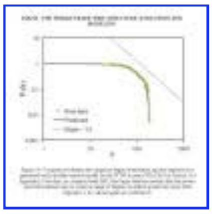

Figure 14. Comparison between the empirical degree distribution and the expected one generated by the hidden-variable model for the WTW in year 1995 (DBII in Section I of Appendix). Note that, as compared with DBI, this larger database reveals that the power-law trend extends only to a narrow range of degrees (a dashed power law curve with exponent -1.6 is shown again as a reference).

Similarly, in Figure 15 we show again the relation between degrees and GDPs in year 1995, with the addition of the predicted trend. Again, experimental results are in accordance with the model.

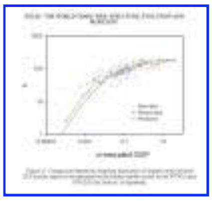

Figure 15. Comparison between the empirical dependence of degrees on the rescaled GDP and the expected one generated by the hidden-variable model for the WTW in year 1995 (DBII in Section I of Appendix).

It is possible to derive higher-order expectations from the model. For instance, the expected average nearest neighbor degree is

$$\langle k_{nn}^{i}(t)\rangle = \frac{\sum_{j\neq i,k\neq j} p_{ij}(t)p_{jk}(t)}{\langle k_i(t)\rangle} \quad (22)$$

and the expected clustering coefficient is

$$\langle c_i(t)\rangle = \frac{\sum_{j\neq k\neq i} p_{ij}(t)p_{jk}(t)p_{ki}(t)}{\langle k_i(t)\rangle^2 - \langle k_i(t)\rangle} \quad (23)$$

The above predictions can be tested against real data by considering again the dependence of k_i^{nn} and c_i on k_i that was shown previously in Figure 4. The comparisons between empirical and model-generated properties are shown in Figure 16, where now the empirical quantities are measured on the database DBII (see Section I of Appendix).

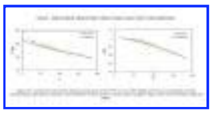

Figure 16. Comparison between the empirical properties of the WTW in year 1995 (DBII in Section I of Appendix) and the corresponding expectations obtained with the hidden-variable model: average nearest neighbor degree (left) and clustering coefficient (right).

Again, the excellent agreement indicates that the model indeed captures the basic aspects of the WTW topology as essentially driven by the GDPs of world countries. The same accordance is found for each annual snapshot of the network in the time interval 1950-2000 (DBII in Appendix).

These results have also another interpretation. Since, as we mentioned, the form of the connection probability used in the above model generates maximally random graphs where only the degree sequence is specified, the surprising prediction of higher-order properties implies that, once the degree sequence is fixed, most of the other properties are automatically explained. In other words, it appears that the complexity of the WTW topology at an undirected and unweighted level relies mainly on the highly heterogeneous distribution of GDPs across world countries. Indeed, this is confirmed by other studies that explicitly considered this problem by randomizing the topology of the WTW while preserving its degree sequence. The randomized versions were found to be almost indistinguishable from the original real-world network. Additional considerations apply to the directed case, that we shall consider in Section 4.2.3. Before that, in the next section we briefly mention a different strategy leading to the same results discussed here.

3.3.1. The Backbone of the WTW 4.2.2. Maximum Likelihood Approach

THE WORLD TRADE WEB: STRUCTURE, EVOLUTION AND MODELING

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Keywords: Economy, International Trade System, World Trade Web, Complex Networks.

Contents

4.2.2. Maximum Likelihood Approach

The results of the previous section are supported and refined by an inverse approach to the extraction of information from the WTW. On a general ground, rather than assuming an empirical quantity as a candidate for the hidden variables $\{x_i\}$ and testing whether the networks generated with this choice are indeed similar to the real-world network considered, one can reverse the strategy and extract the values of the hidden variables $\{x_i\}$ directly from the real network. Then one can compare these unique values with candidate empirical quantities, to check whether a relation really exists. This comparison would automatically provide the form of the dependence between $\{x_i\}$ and the empirical quantities.

A way to realize this approach is provided by the Maximum Likelihood (ML) principle, a procedure commonly used in statistics whose use can be easily extended to networks. The ML principle has been shown to indicate a unique, statistically correct choice of parameters for network modeling, ruling out subjective criteria. The main point is that, rather than requiring the values of model parameters as the input and generating an ensemble of possible networks as the output, the ML principle requires one particular real-world network as the input and provides the corresponding optimal parameter values as the output. "Optimal" stands for the parameter values that maximize the likelihood (or equivalently its logarithm) to obtain the particular real-world network under the model considered.

As discussed above, if one is interested in a null model where only the degree sequence is specified, then the correct model is one where a link between vertices i and j is drawn with probability $p_{ij} = \delta x_i x_j / \left(1+\delta x_i x_j\right)$. The corresponding probability to generate a real-world network with adjacency matrix **A** is

$$P(\mathbf{A}) = \prod_{i<j} p_{ij}{}^{a_{ij}} (1-p_{ij})^{1-a_{ij}} = \prod_{i<j} \left(\frac{p_{ij}}{1-p_{ij}}\right)^{a_{ij}} (1-p_{ij})$$

(24)

where a_{ij} correspond to the elements of the matrix **A**. The logarithm of $P(\mathbf{A})$ is the "log-likelihood" λ to generate the network. For the particular form of p_{ij} considered, it reads

$$\lambda(\delta, x_1,\dots,x_N) = \log P(\mathbf{A}) = \sum_{i<j} a_{ij} \log(\delta x_i x_j) - \sum_{i<j} \log(1+\delta x_i x_j)$$

(25)

where the dependence of λ on the parameters $\{\delta, x_1,\dots,x_N\}$ of the model has been made explicit. The particular values $\{\delta^*, x_1{}^*,\dots,x_N{}^*\}$ of these parameters that maximize λ are easily found by taking derivatives, which leads to the following $N+1$ coupled equations (to be satisfied for all countries $i=1,\dots,N$):

$$L = \sum_{i<j} \frac{\delta^* x_i^* x_j^*}{1+\delta^* x_i^* x_j^*} \quad k_i = \sum_{j\neq i} \frac{\delta^* x_i^* x_j^*}{1+\delta^* x_i^* x_j^*}$$

(26)

where $L = \sum_{i<j} a_{ij}$ is the total number of links in the real network, and

$k_i = \sum_j a_{ij}$ is the degree of vertex i in the same network. The above equations fix the optimal values $\left\{\delta^*, x_1^*, \ldots, x_N^*\right\}$ of both the parameter δ and the hidden variables $\{x_i\}$ of each vertex i. Note however that the expression on the left is automatically verified once the right is satisfied for all i, which simply reflects the fact that the parameter δ, otherwise undetermined, can be always reabsorbed in the parameters $\{x_i\}$.

The above considerations imply that the N equations on the right are the only independent equations. Note that this set of equations is formally similar to the formula, shown in the previous section, expressing the expected degree $\langle k_i \rangle$ as a sum over the probabilities $p(x_i, x_j)$ in the hidden variable model. However the two expressions have opposite meanings. In the hidden variable model the empirical input quantities are the rescaled GDP values $\{x_i\}$, which are fixed by observation, while the output values are the expected degrees $\{\langle k_i \rangle\}$. These expected values, and any other expected topological property, can then be compared with (but not fitted to) the empirical values $\{k_i\}$. By contrast, in the maximum likelihood approach the empirical input quantities are the degrees $\{k_i\}$ while the $\left\{x_i^*\right\}$ are output values depending uniquely on the observed degree sequence. In this case too, these output values can be compared with (but not fitted to) the empirical rescaled GDPs $\{x_i\}$. This comparison in shown in Figure 17, where for consistency the parameter δ^* used in the hidden variable model and the same parameter used in the maximum likelihood approach have been both reabsorbed in the variables $\{x_i\}$ by redefining the latter as follows: $x_i \to x_i\sqrt{\delta}$. One finds that the fitness values determined using only topological information are indeed proportional to the empirical GDPs of world countries, and therefore that the maximum likelihood approach successfully identifies the GDP as the “hidden” variable shaping the topology of the WTW. Note that the two sets of values are in principle completely independent. The ML approach allows to test alternative hypotheses about different candidates expected to explain the topology of world trade by simply comparing any alternative empirical quantity with the set of values $\left\{x_i^*\right\}$, which is instead unique. Similarly, if the chosen quantity is indeed shaping the network, but is not simply proportional to the hidden variables, then the comparison with the latter would automatically indicate the form of the dependence. By contrast, the hidden variable model used in its “direct” form requires separate analyses for each candidate quantity and each expected relation between it and the hidden variables. This makes the use of the ML approach preferable.

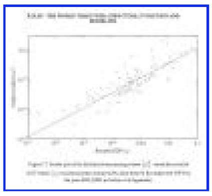

Figure 17. Scatter plot of the likelihood-maximizing values $\{x_i^*\}$ versus the rescaled GDP values $\{x_i\}$ (isolated points) and linear fit (solid line) for the undirected WTW in the year 2000 (DBII in Section I of Appendix).

4.2.1. Hidden Variable Model 4.2.3. Directed Models

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THE WORLD TRADE WEB: STRUCTURE, EVOLUTION AND MODELING

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Contents

4.2.3. Directed Models

As we discussed in Section 3, the WTW displays a particular directed structure. The directed properties of the network depend on the representation adopted, i.e. on whether links are considered unweighted, weighted by trade volumes, or weighted by trade imbalances. The gravity models described in Section 4.1 allow for a directed weighted description where the fluxes w_{ij} and w_{ji} are different, as the two exponents β_1 and β_2 can take different values. However, as already discussed, gravity models do not predict zero trade flows. In a directed scenario, this also implies that they generate fully reciprocated networks where a pair of links pointing in opposite directions is always present between any two vertices. This means that the reciprocity parameter introduced in Section 3.1 is strictly $r = 1$, in contrast with the empirical values corresponding to any representation of the WTW ($r < 1$ when bilateral trades are considered, and $r = 0$ when trade imbalances are considered). This is an additional drawback of gravity models, inherent to a directed network description. By contrast, as we now show hidden variable models can be extended in such a way that directed webs with any value of the reciprocity are reproduced.

The simplest generalization of the hidden variable model to the directed case consists in assigning each vertex i two fitness values $\{x_i, y_i\}$, separately controlling the tendency to form outgoing ad incoming links respectively. This means that each pair of vertices i, j must be considered twice according to the two possible directed links that can be drawn between them (from i to j and then from j to i). The probability that a directed link from i to j exists is specified by a function $p_{ij} = p\left(x_i, y_j\right)$, which now is in general not symmetric, i.e. $p\left(x, y\right) \neq p\left(y, x\right)$. Note however that in such a way the probability that a link from i to j is drawn is independent on whether the reciprocal link from j to i is there. As can be easily shown, this implies that the coefficient ρ of reciprocity introduced in Section 3.1 is zero. Therefore, this simple directed extension of the hidden variable model to directed networks does not reproduce the non-trivial reciprocity structure observed in the WTW and in other real networks.

In order to generate networks with non-trivial reciprocity, one must introduce a mechanism such that the probability that a link from i to j is drawn depends on whether the reciprocal link from j to i is there. This can be achieved by assigning different probabilities to the four possible events that can occur between any two vertices i and j. In particular, one can separately specify the probabilities of having two mutual links between i and j, a single link from i to j (and no reciprocal link from j to i), a single link from j to i (and no reciprocal link from i to j), and no link in either direction. Clearly, these four probabilities must sum up to one since they cover all the possible outcomes. The network can then be generated by drawing, for each single vertex pair, a single link from i to j, a single link from j to i, two mutual links, or no link at all with the corresponding probabilities. As a result, in this hidden variable model each vertex is in general assigned three fitness values $\{x_i, y_i, w_i\}$, separately controlling its tendency to form non-reciprocated outgoing links, non-reciprocated incoming links, and reciprocated links going both ways, respectively. In the particular case of the WTW, however, the results described in Section 3.1 imply that the empirical reciprocity structure of international trade displays a somewhat simplified form. In particular, it is possible to show that the three values { $\{x_i, y_i, w_i\}$ are all again related to the GDP alone. This allows one to derive simultaneous expectations for the in-degrees $k_i^{\text{in}} = \sum_j a_{ji}$, out-degrees $k_i^{\text{out}} = \sum_j a_{ij}$ and reciprocal degrees

$k_i^{\leftrightarrow} = \sum_j a_{ij} a_{ji}$. For instance, the probability that at time t a directed link exists from country i to country j takes the simple form

$$p_{ij}(t) = \frac{\alpha(t) x_i(t) x_j(t)}{1 + \beta(t) x_i(t) x_j(t)} \tag{27}$$

where as in Section 3.1 $x_i(t)$ is the GDP rescaled to the average value in the same year t, from which one can derive the expected in- and out-degree of vertex i. According to the maximum likelihood principle, the parameters $\alpha(t)$ and $\beta(t)$ can now be fixed by imposing that the expected total number of links and the expected number or reciprocated links simultaneously match their empirical values. These two parameters have a direct dependence on the reciprocity $r(t)$, and it can indeed be confirmed that the optimal values of $\alpha(t)$ and $\beta(t)$ obtained under the ML principle are in close agreement with the corresponding estimates obtained exploiting the observed value of $r(t)$. As in the undirected case, the observed topological properties of the WTW that explicitly involve its reciprocity structure are excellently reproduced by this extended hidden variable model, highlighting once again that the GDP is the main factor underlying the shape of the international trade network.

5. Conclusions and Some Open Questions

The complex network approach to the understanding of the international trade system has proved to provide valuable insights into its structure and evolution. The use of the trade network representation has enabled the consideration of non-local effects in the analysis of trade interdependencies that has revealed unexpected potential effects of one economy on another. At the modeling level, we are now in the position of understanding some of the key variables, such as the GDP, that explain the observed WTW topology. Nevertheless, many questions remain to be explored in depth. We point out four of them as examples of feasible directions of research in the near future.

Community structure: Between the large global scale of the complete international trade system and the microscopic scale of isolated interactions, a mesoscopic organization is also present in the WTW. The study of this mesoscopic scale, and more specifically of the WTW community structure, can greatly benefit from a weighted network representation. In the unweighted approach, communities are customarily defined as groups of nodes that have a dense set of edges among each other, but a relatively few number of edges to other densely connected subsets. While the high density in number of connections of the unweighted WTW hides any information about the organization of the network in communities, the heterogeneity in the weights and strengths of the weighted version can be exploited to identify more easily the existence of differentiated groups. In this respect, there have been recent attempts to identify trade communities. Nevertheless, community detection in such densely connected systems is extremely difficult and sensitive to the method employed. Further developments are thus necessary.

Propagation of economic crises at the world scale: In a globalized economy, we face ever-increasing problems in disentangling the complex set of relations and causality that might lead to crisis or increased stability. Focusing on just the bilateral relations among country economies is a reductionist approach that cannot work in a highly interconnected complex system. Future work on these grounds might help in the assessment of world trade relations and the understanding of the global dynamics underlying major economic crises.

Interplay between dynamics and topology: As highlighted by the success of both gravity models in explaining trade volumes and hidden variable models in explaining trade connectivity at various levels of organization, the structure of the WTW is strongly affected by the GDP of world countries. On the other hand, the aggregated value of all bilateral trade flows for a certain country in its turn affects GDP levels. The GDP of a country is defined as the market value of all final goods and services produced within its borders in a given period of time. In the expenditure based approach, it is decomposed in several terms as $GDP(t) = C(t) + I(t) + G(t) + F(t)$, where $C(t)$ stands for private consumption, $I(t)$ for business investments in capital, $G(t)$ for government spending and $F(t)$ for net trade balance. So, internal contributions are corrected by the trade interactions with other countries. This mutual dependence leads to the intriguing picture of a continuous interplay between the dynamics and topology of the WTW, a scenario that intimately relates the results of network theory to those of economic modeling. The sudden transition in the relation between net trade-flows and total GDP in the 1960s has been tied up, from that moment, with their evolution. Although not a proof, this seems to suggest that the internal components of the economies (private, business and government spending) become more dependent on trade exchanges with other countries, and this leads to the conjecture that GDP and international trade, or in other words internal and external components, are entangled in a complex continuous feedback mechanism. Nevertheless, the integration of markets in the last decades does not seem to come in a smooth gradual transformation but rather like a fault transition. Whether this is indicative of the birth of a truly global market where all the economies are effectively interwoven beyond trade needs more proof. New validations about the reach of the phase transition should be performed as well. One example of a technical question that immediately arises is whether the empirical success of gravity models for explaining bilateral trade as a function of GDP after 1960 is maintained when studying historical data before this date. Thus, to understand better how GDP and trade imbalances are related, the empirically successful gravity model of international trade needs to be revisited. Similarly, gravity models need to be reconciled with hidden variable models within a single and more general framework, as one currently needs both models to predict trade volumes and trade connections separately.

Commodity-specific analyses: All the results reported here are based on representations of the international trade network where a single trade interaction (whether weighted or unweighted, directed or undirected) aggregates the information about all types of products and goods that are traded between two world countries. However, even interactions that are undistinguishable on an aggregated basis (for instance because they have an equal weight) may reflect very different trade patterns, if the products involved are different. Thus, a more detailed description of the trade system requires the nature of the commodities being exchanged be taken into account. Technically, this amounts to regard the aggregate network as the superposition of different networks, each referring to a specific commodity. The number of commodities involved depends on the level of resolution reported in the available databases. Studying commodity-specific trade within a complex network framework would allow to answer a series of new questions, and to push the understanding of the trade system to an unprecedented depth. Very recent results have started to address this problem, but much more has to be done in the future.

In one study, a “product space” where goods involved in similar trade patterns are located nearby has been reconstructed. It has been found that less sophisticated products are located in a sparsely connected periphery, while more sophisticated products occupy a denser core. Poor countries are more concentrated in exports of peripheral goods, while rich countries are more concentrated in core goods. Moreover, countries are empirically found to grow by developing new products similar to the ones they currently produce, corresponding to small movements across the product space. The observed core-periphery structure of the latter implies that, in order to develop more sophisticated products, poor countries must traverse longer distances. This is one of the possible reasons why poor countries face higher barriers in developing more competitive exports, and consequently experience troubles in converging to the income levels of rich countries.

Another study has focused explicitly on the relationship between the aggregate trade network and its commodity-specific components. It has been found that the approximate log-normality of the link weight distribution observed in the compound network is generated as an outcome of aggregation, as link weight distributions of commodity-specific networks are extremely heterogeneous. Similarly, many properties of commodity-specific trade networks, such as average connectivity, clustering and centrality levels, are very different from their aggregate counterparts. Moreover, a surprisingly important role is played by weak commodity-specific links, as these links are responsible for the connectedness of commodity-specific networks, which in turn results in the connectedness of world trade at an aggregate level. The analysis of commodity-specific networks also allows to detect the dynamics of trade preferences between countries empirically. In principle, one can expect the following two opposite tendencies. In case of common geographic borders, trade agreements, or membership to the same free trade association or currency union, two countries may prefer to exchange various different types of commodities even if there are many potential alternative trade partners, either as importers or as exporters, for each commodity. Conversely, in presence of specialized trade preferences, pairs of countries may tend to have focused exchanges mainly involving a small set of commodities. It is possible to analyze the actual evolution of trade preferences by considering the dynamics of correlations between commodity-specific networks. Empirical results suggest that the enhancement of trade specialization dominates over the opposite tendency, and that pairs of countries have been developing more and more commodity-intensive trade relationships characterized by a decreasing variety of goods. This means that the roles played by different commodities in the international trade system have become more and more dissimilar.

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Appendix

Appendix

Related Chapters

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Glossary

Backbone	**:** The smallest subgraph containing the relevant information of a given network. The "relevant information" may differ in different contexts and so the definition of the backbone.
Complex network	**:** Topological representation of the patterns of interactions observed in the majority of complex systems. Units are represented as nodes or vertices and interactions as links or edges connecting those nodes.
Complex system	**:** System composed of interacting parts displaying emergent properties as a result of these interactions.
Degree correlations	**:** Statistical correlations between the degrees of pairs of connected nodes.
Degree distribution	**:** Distribution of node's degrees for the whole network.
Disparity	**:** Measure of the local heterogeneity of the weights of a given node.
Gross Domestic Product	**:** The total market value of all final goods and services produced in a country in a given year, equal to total consumer, investment and government spending, plus the value of

(GDP) exports, minus the value of imports.

Link weight : Intensity associated with a connection between two nodes.

Logarithmic binning : A way to filter noise from power-law distributed data by averaging observations within bins that are equally spaced in logarithmic scale.

Maximum Likelihood principle : A key approach in statistical theory where the free parameters of a model are set to the 'optimal' values that maximize the probability of observing the available empirical data under the model itself.

Node degree : Number of connections or neighbors of a given node.

Node strength : Sum of the intensities of all the connections of a given node.

Null model : Statistical reference model that sets the baseline for the detection of intrinsic properties. The null model accounts for what can be expected by chance or for structural constraints.

Reciprocity : The tendency of pairs of vertices in a network to establish two mutual connections in opposite directions.

World Trade Web (WTW) : Complex network of trade relationships between world countries.

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4.2.2. Maximum Likelihood Approach